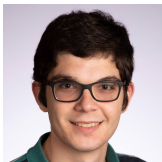




# Online Team Formation under Different Synergies

**Matthew Eichhorn**, Sid Banerjee, David Kempe

WINE 2022



Paper (Extended): <https://arxiv.org/abs/2210.05795>

Team formation is ubiquitous in many domains

- Online Education/Labor Platforms
- Projects need multi-agent teams
- “Success” of team depends on synergy of members
- Individuals' attributes that affect synergy are unknown



How should we group agents to efficiently find an optimal partition?

# Setting

**Agents:**  $n$  individuals with **unknown** type  $\theta_i \in \{0, 1\}$

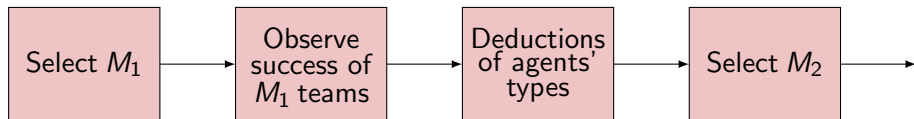
- $k =$  number of type-1 agents (**unknown**)
- $n, k$  even

**Teams:** Each team comprises 2 agents

- Success of team  $\{i, j\}$  given by **known** symmetric synergy function  $f: \{0, 1\}^2 \rightarrow \mathbb{R}$

**Rounds:** In round  $t$ , principal selects matching  $M_t$

- Observes  $f(\theta_i, \theta_j)$  for each  $(i, j) \in M_t$



# Objective

Score:  $S(M) = \sum_{(i,j) \in M} f(\theta_i, \theta_j)$

Optimum:  $M^* = \operatorname{argmax}_M \{S(M)\}$

Goal: Select matching policy to minimize *Total Regret*

$$\sum_{t=1}^{\infty} (S(M^*) - S(M_t)).$$

- Agents' types selected by an *adaptive* adversary
- Randomization will not help
- Optimal policy depends on team synergy function  $f$

# Reducing to Boolean Synergy Functions

- $f$  is completely described by  $f(0, 0)$ ,  $f(0, 1) = f(1, 0)$ ,  $f(1, 1)$
- Regret is linear in number of each type of team played

## Lemma

*Total regret under any symmetric  $f$  is determined by regret achieved under **boolean** synergy functions  $\{EQ, XOR, OR, AND\}$ .*

# Main Results

- All bounds are non-trivial and instance ( $k$ ) dependent
- Algorithms are agnostic of  $k$  but achieve near-optimal regret for all  $k$

## EQ

Matching Lower/Upper Bound:

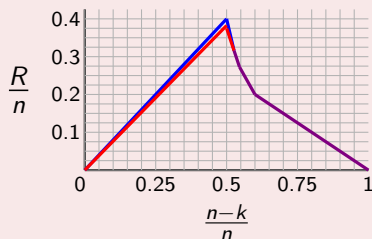
$$R = 2 \cdot \min(k, n - k)$$

## XOR

Matching Lower/Upper Bound:

$$R = 2 \cdot (\min(k, n - k) - 1)^+$$

## OR



## AND

Lower Bound:

$$R \geq n - k$$

Upper Bound:

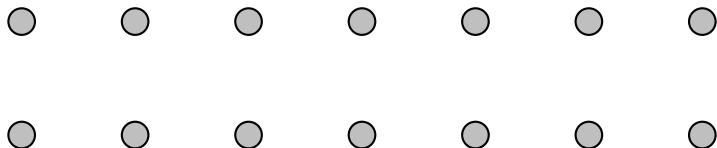
$$R \leq n - k + \left\lfloor \frac{\min(k, n - k)}{4} \right\rfloor$$

# The EQ synergy function

- $f(0,0) = f(1,1) = 1$     $f(0,1) = 0$
- Promotes cohesive teams
- Optimal matching has as few  $(0,1)$  teams as possible
- Can find  $M^*$  in 3 rounds with a “local search” strategy

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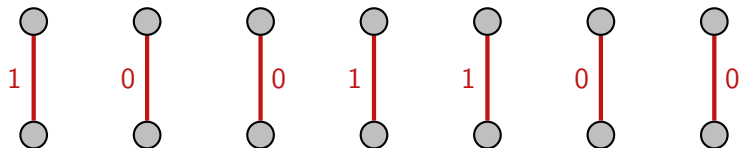
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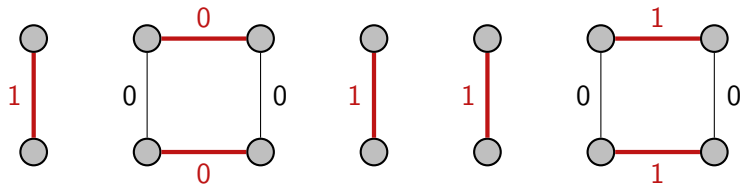
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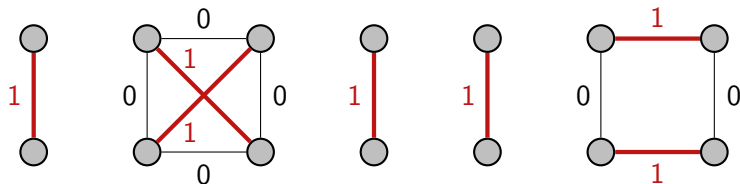
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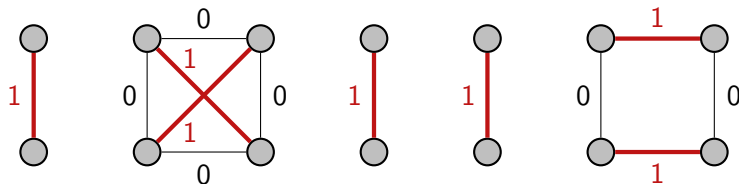
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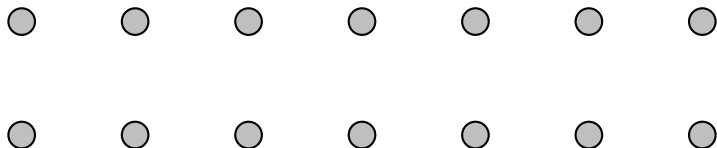
$$\text{Total Regret} \leq 2 \cdot \min(k, n - k)$$

# The XOR synergy function

- $f(0,0) = f(1,1) = 0$     $f(0,1) = 1$
- Promotes diverse teams
- Optimal matching has as many  $(0,1)$  teams as possible
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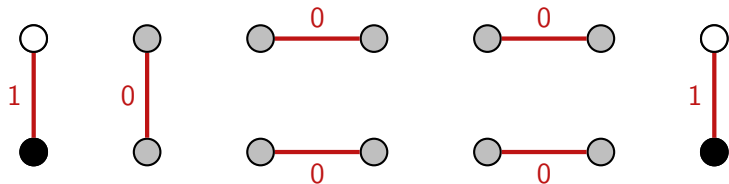
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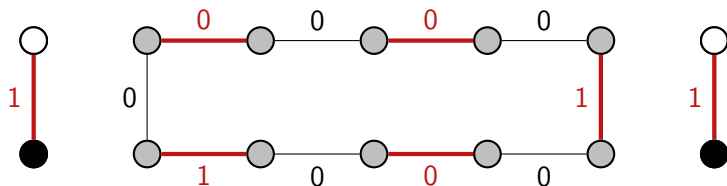
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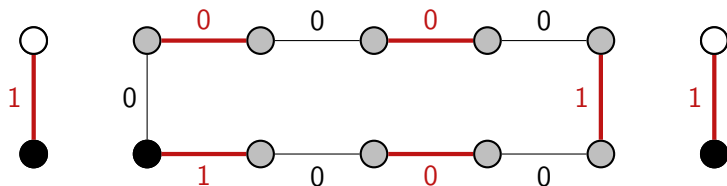
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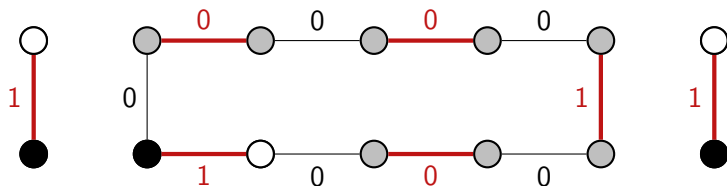
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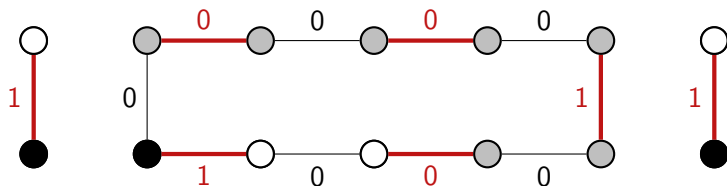
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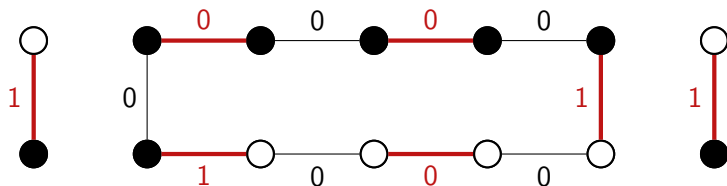
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# The OR synergy function

- $f(0,1) = f(1,1) = 1$      $f(0,0) = 0$
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- Some  $(0,0)$  teams revealed in Round 1
- Locate  $(1,1)$  teams to re-partner with the  $(0,0)$  teams
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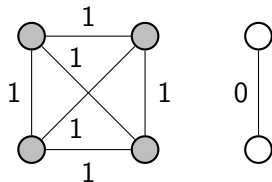


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- Can't distinguish  $(1, 1)$  teams from  $(0, 1)$  teams
  
- Use known 0-agents to “explore” types of unknown agents
- Can be slow, incur too much regret

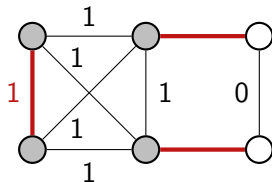
# Key Algorithmic Insight

4-cliques of successful teams can quickly “neutralize” unsuccessful teams.



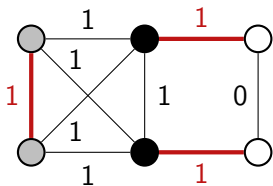
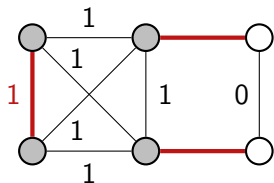
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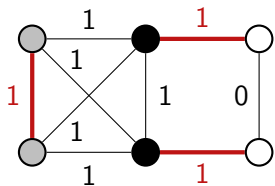
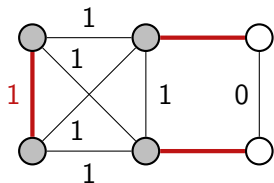
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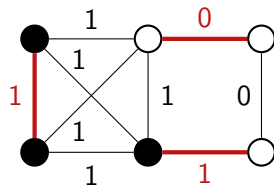


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or

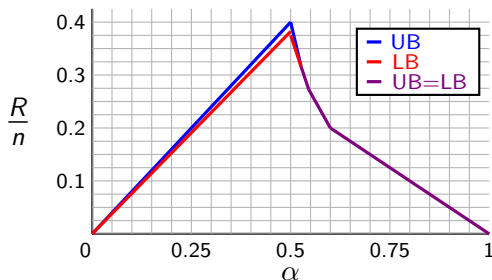


# Max Exploit w/ 4-Cliques

**Algorithm Idea:** Explore with known 0-agents while re-pairing successful teams to form 4-cliques.

## Regret Guarantee

- Parameterized by  $\alpha = \frac{n-k}{k}$ , fraction of low-skill agents
- Analysis is involved; different techniques for different ranges of  $\alpha$
- Nearly-matching upper/lower bounds that agree for  $\alpha \geq \frac{10}{19}$



# The AND synergy function

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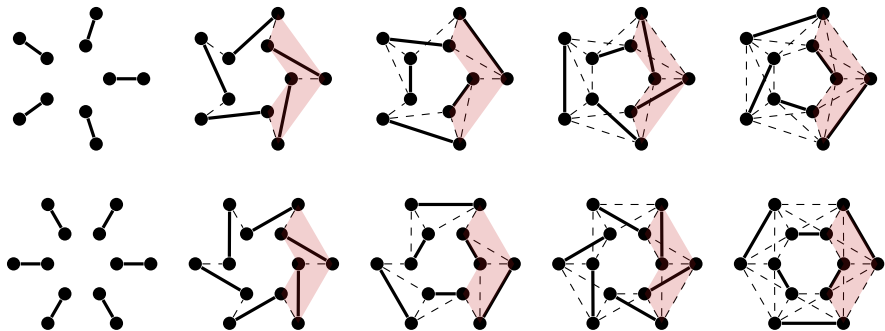
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- **Idea:** “Charge” regret to 0-agents when they pair with 1-agent
- Place agents in big ring, pair with neighbors at increasing distances
- Before 0-agent pairs with 3 1-agents, two will have been paired up



# RING FACTORIZATION w/ REPAIRS

- Must use “double ring” structure to deal with parity issues
- Case-specific repair step when (1,1) teams discovered



- Total Regret  $\leq n - k + \left\lfloor \frac{\min(n-k, k)}{4} \right\rfloor$

- Can the gaps in our regret bounds be closed/tightened?
- We assume perfect feedback. What if signal of team success is noisy?
- How can we handle larger teams?

# Thanks for Listening

For more details, please see our paper!

