MATH 2210 Name: _____

Workshop 1: Balancing Chemical Reactions ID: _____

Work in your group to complete the following exercises. You may print this handout, annotate the PDF or write your answer on paper. Make your grader's life easier by writing neatly and legibly!

Please include full explanations and write your answers using complete sentences (not just a bunch of mathematical symbols!). It is important to be able to explain your reasoning to someone else in writing.

Warmup

Question 1.

(a) Represent the following homogeneous system as an augmented matrix.

$$\begin{aligned} 2w &= y \\ 6w &= 2z \\ 2x &= 2y + z \end{aligned}$$

(b) Does this system have a non-trivial solution? How do you know?

(c) Find a representation of the general solution of this system.

Application: Balancing Chemical Reactions

In this workshop, we'll explore how the techniques that we have developed for solving homogeneous linear systems can be applied to balance chemical reactions. A reaction equation shows the reagents (inputs) and products (outputs) of a reaction. For example, the equation,

$$2\,H_2 + O_2 \rightleftharpoons 2\,H_2O$$

describes how hydrogen and oxygen gas react to form water vapor. The coefficients on each molecule are all positive integers, chosen so that the number number of atoms of each element is preserved by the reaction. In the above example, there are 4 hydrogen and 2 oxygen atoms on each side of the equation. Determining coefficients is critical because it gives insight into which reactions are possible, and it helps chemists to determine the optimal amounts of each reagent to use in their reactions.

Question 2. Consider the following chemical reaction with undetermined coefficients:

$$\_\_\,Al_2O_3 + \_\_\,Fe \longrightarrow \_\_\,Fe_3O_4 + \_\_\,Al$$

(a) Introduce four variables for each of the four coefficients. Then, write an expression equating the number of aluminium (Al) atoms on each side of the reaction equation.

Repeat this for the oxygen (O) and iron (Fe) atoms.

(b) Transform these equations into a homogeneous system, and represent it with a matrix of coefficients.

(c) Determine the general solution for this system.

(d) Scale the general solution to determine the solution vector for which all of the coefficients are positive integers. Use this to write the balanced chemical equation.

Question 3. The combustion of benzanoic acid (used for the creation of some common food preservatives) is represented by the reaction equation,

$$\_\_\,C_7H_6O_2 + \_\_\,O_2 \longrightarrow \_\_\,H_2O + \_\_\,CO_2$$

Determine the smallest integer coefficients that balance this reaction.

Question 4. A chemistry student is trying to understand the formation of acid rain. They learn that factory emissions of nitrogen oxides can lead to the formation of nitric acid (HNO_3) in the atmosphere, and suggests modeling this with the chemical reaction,

$$\_\_\,NO_2 + \_\_\,H_2O \longrightarrow \_\_\,HNO_3$$

Using the process from above, determine whether this is a sensible reaction mechanism.

Question 5. Scientists have observed that the reaction

$$\_\_\,\mathrm{CO} + \_\_\,\mathrm{CO_2} + \_\_\,\mathrm{H_2} \longrightarrow \_\_\,\mathrm{CH_4} + \_\_\,\mathrm{H_2O}$$

occurs in different proportions (that is, with different coefficients) depending on environmental conditions. Find the general solution of the system of equations given by this reaction. Use this to write at least two balanced reaction equations (that are not multiples of each other).

Question 6. Argue that although there is a non-trivial solution to the system given by the reaction equation,

$$\_\_\,\mathrm{MnO_2} + \_\_\,\mathrm{K_2CO_3} + \_\_\,\mathrm{KNO_2} \longrightarrow \_\_\,\mathrm{K_2MnO_4} + \_\_\,\mathrm{KNO_3} + \_\_\,\mathrm{CO_2},$$

this reaction cannot occur.