Recitation 9

1. Consider the population model with crowding given by

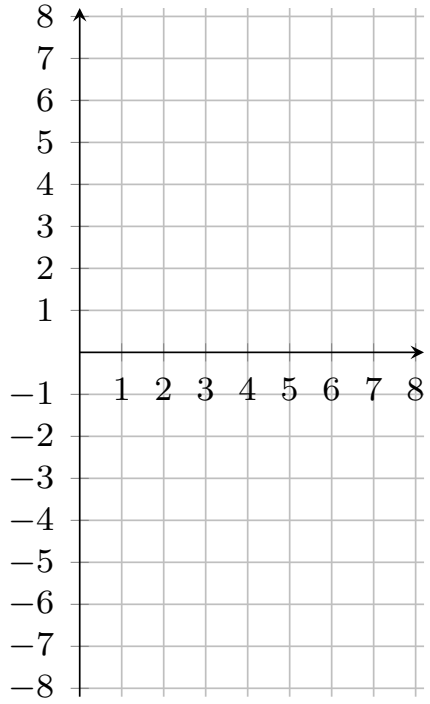
$$X' = 0.5X(1 - \frac{X}{100}).$$

Use Euler's method with $\Delta t = 0.2$ to approximate the population at $t = 0.6$, assuming that the initial population is $X = 25$.

2. Consider the mass-spring model without friction given by

$$X' = V$$
$$V' = -X$$

(a) Starting at the point $(4, 4)$, draw an estimate trajectory for the system between $t = 0$ and $t = 2$ using Euler's method with a step size $\Delta t = 0.5$. Then repeat the same process but for the trajectory of the system between $t = 0$ and $t = 0.4$ and using a step size of 0.1.



(b) How does the precise trajectory look like for this system (this has appeared in the reading)? Which of the two approximations is the most accurate?

3. Possible Issue with Euler's Method:

(a) Consider the shark-tuna model given by $T' = T - ST$ and $S' = ST - S$. Let us start with 8 tuna and 2 shark. Compute 3 steps of Euler's method using $\Delta t = 0.5$.

(b) What is wrong with the previous computations? How coul we fix this?

4. Summary

(a) What is the purpose of Euler's method and why is it useful?

(b) What are the pros and cons of using a smaller Δt?