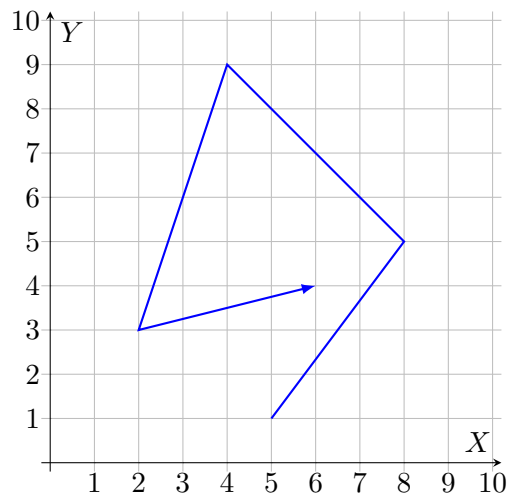


Recitation 7

1. Consider the following trajectory.



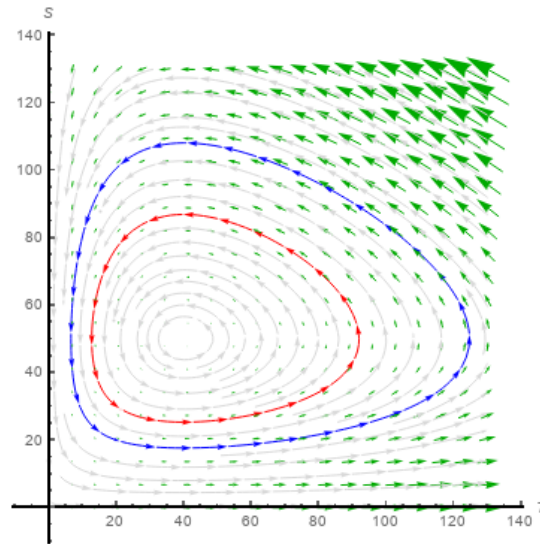
Draw an associated time series.



2. In this question, we consider the shark-tuna system governed by the equations.

$$\begin{aligned}T' &= 0.5T - 0.01ST \\S' &= 0.005ST - 0.2S\end{aligned}$$

Two trajectories for this system are superimposed on its vector field in the following picture.



We consider the process of sketching the time series associated with these trajectories.

(a) First, consider the blue trajectory with initial point $P_0 = (75, 25)$. We reason about the value of T along this trajectory.

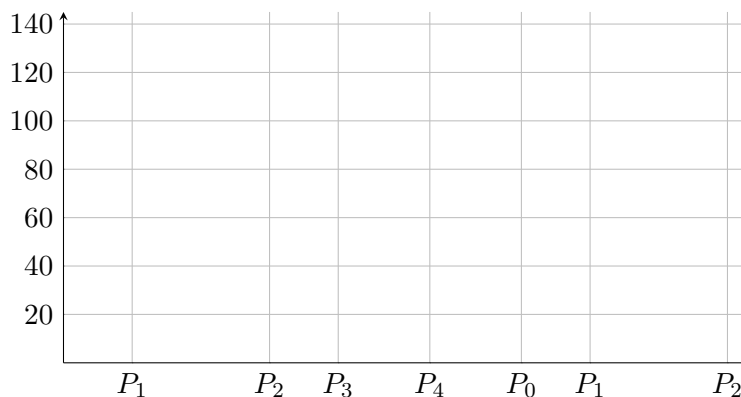
1. Initially, T is increasing.
2. What is the maximum value of T that the trajectory reaches?
3. Label this point on the trajectory P_1 . What is the value of S at P_1 ?
4. Now, T begins to decrease.
5. What is the minimum value of T that the trajectory reaches?
6. Label this point on the trajectory P_3 . What is the value of S at P_3 ?
7. Finally, T increases again toward P_0 , completing one cycle of the trajectory.

Now, we reason about the value of S along this trajectory.

1. Initially, S is increasing.
2. What is the maximum value of S that the trajectory reaches?
3. Label this point on the trajectory P_2 . What is the value of T at P_2 ?
4. Now, S begins to decrease.
5. What is the minimum value of S that the trajectory reaches?

- 6. Label this point on the trajectory P_4 . What is the value of T at P_4 ?
- 7. Finally, S increases again toward P_0 , completing one cycle of the trajectory.

(b) Put together the information you gathered in part (a) to sketch the time series associated with the blue trajectory.



(c) Repeat this process to sketch the time series associated with the red trajectory with initial point $(80, 35)$.



(d) What do these time series have in common? What is the main difference between these time series?

3. Consider a two-variable system whose trajectory is a single point. Sketch its trajectory in the state space and its associated time series.