

Recitation 6

1. Match the six pairs of change equations to their corresponding vector fields. Note that the vectors have been **normalized**. Explain how you made each of these choices. (Hint: Compute the change vectors at some chosen points. Try to determine where the change vectors are horizontal/vertical.)

$$\begin{aligned} X' &= X + Y \\ Y' &= -1 \end{aligned}$$

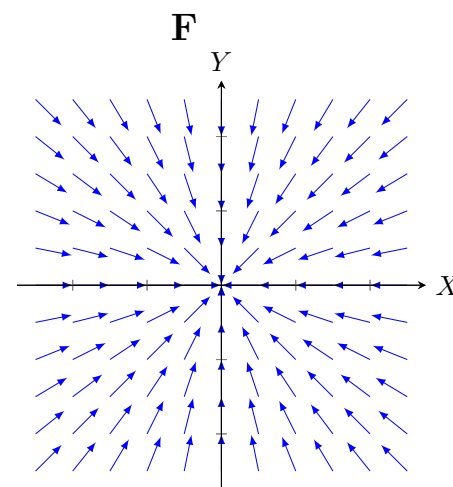
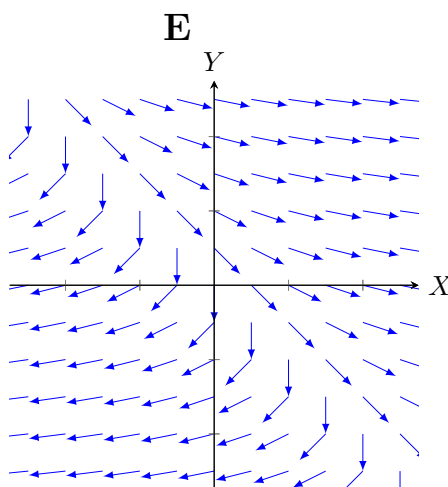
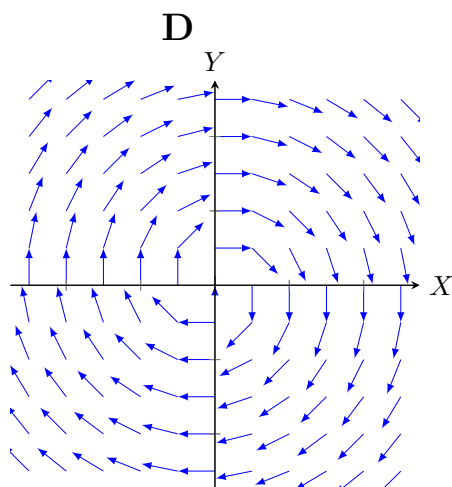
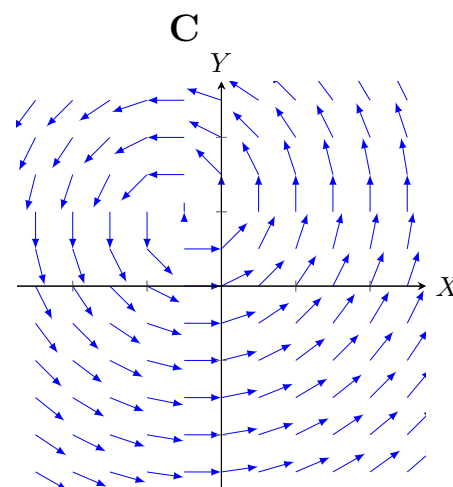
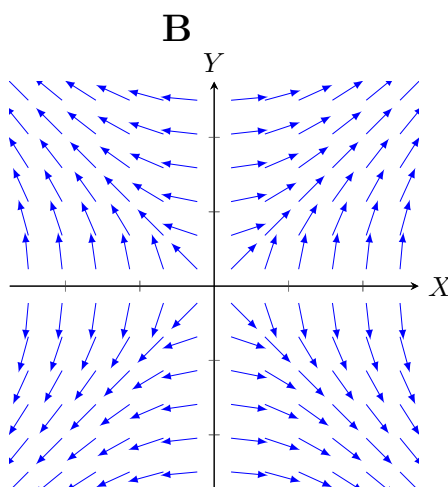
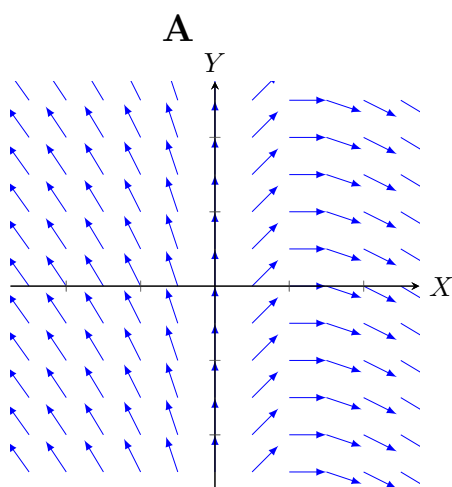
$$\begin{aligned} X' &= -X \\ Y' &= -Y \end{aligned}$$

$$\begin{aligned} X' &= Y \\ Y' &= -X \end{aligned}$$

$$\begin{aligned} X' &= 2 - Y \\ Y' &= X + 1 \end{aligned}$$

$$\begin{aligned} X' &= X \\ Y' &= 2 - X \end{aligned}$$

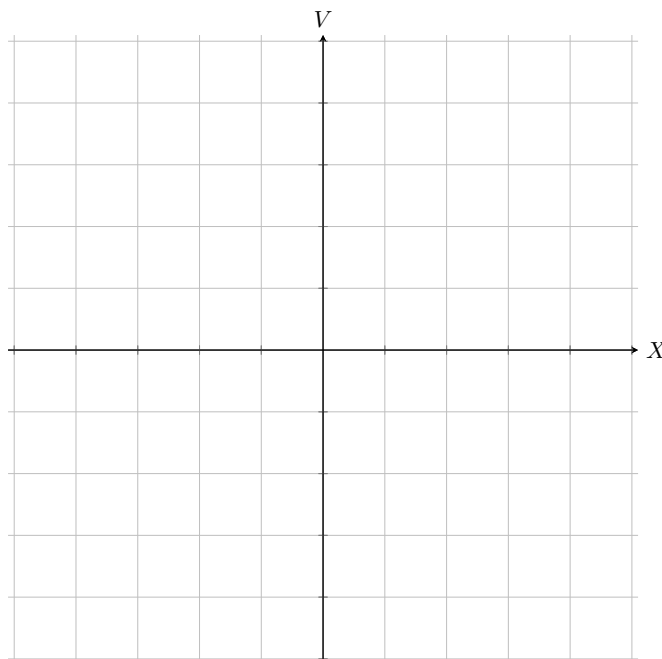
$$\begin{aligned} X' &= X^{-1} \\ Y' &= Y^{-1} \end{aligned}$$



2. In this exercise, we will draw the vector field for the mass-spring system, and see how it changes when a friction term is introduced.

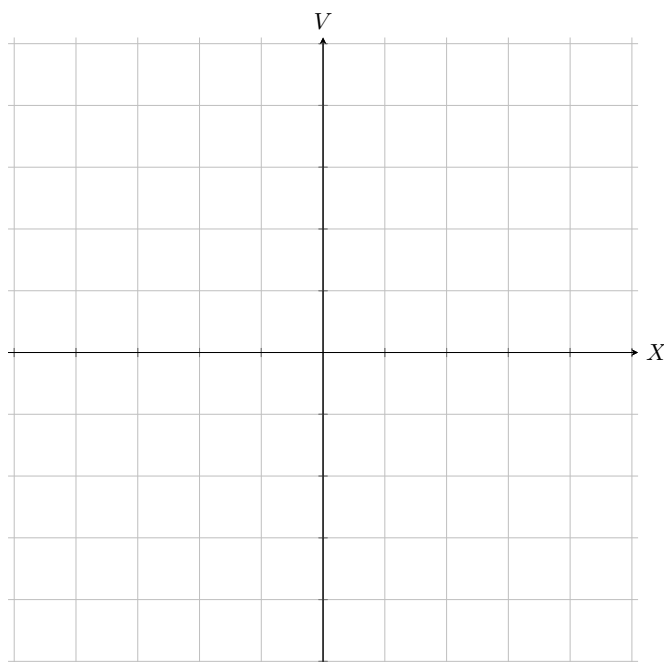
(a) First, we consider the system without friction. As review (that is, try answering this question from memory before consulting your notes or group mates), what are the change equations? What do each of the variables and coefficients represent, and what are their units?

(b) In order to make the computations easy, suppose that $m = k = 1$. Compute the change vectors for 8 different points, choosing at least one point from each quadrant and some points along the coordinate axes. Then draw the vectors on the graph below.



- (c) Now, we turn our attention to the system with friction (proportional to and opposing the velocity). As review, write down the change equations and describe the significance of the new terms and coefficients.

- (d) Again, simplify the equations by setting all constants equal to 1. Compute the change vectors at the same points as in part (b) and draw these vectors on the graph below.



- (e) Finally, we consider the mass-spring system with *negative* friction. In this situation, an external force, or impulse, is acting on the spring in the same direction that it is moving. Write the change equations for this system (under the simplifying assumption that all coefficients are 1). Then, without calculating it, describe what the vector field will look like for this negative friction system.