Recitation 5

1. A simple model of infectious disease spread is

$$U' = sW - mU - pU$$
$$V' = qV - mV - rVW$$
$$W' = rVW - mW - sW$$

where s, m, p, q, and r are positive constants. The variables U, V, and W stand for susceptible population, infected (but not symptomatic) population, and symptomatic population (but not necessarily in that order).

(a) Draw a compartmental model for this system.

(b) Explain in words which variable correspond to which population. How did you determine this?

(c) Provide an interpretation of each of the coefficients (s, m, p, q, r) in the model.

2. Compartmental Models with Vaccination:

(a) Imagine we have a SIR model for measles. People who recover from measles gain permanent immunity. Draw the box-and-arrow diagram for this model and write down coefficients as appropriate.

(b) Now imagine that we model vaccination. How would your model change? Draw the box-and-arrow diagram and write down coefficients as appropriate.

(c) Now, imagine that we find out that the vaccine is "leaky", meaning that some vaccinated people can still get the disease. How do we modify the box-and-arrow diagram to account for this?

(d) Adapt the diagram from part (a) to account for natural births and deaths, as well as disease-induced death.

(e) In the case of some contagious diseases, an infected mother will give birth to an infected child. Modify the diagram from part (d) to account for this.