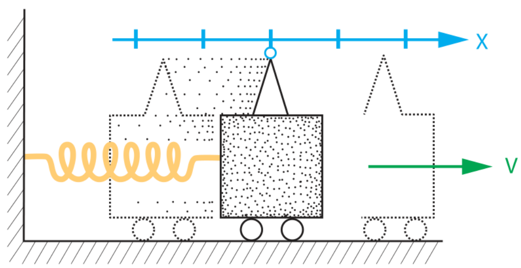
Recitation 4

1. Complete the *Modeling Disease Transmission* activity on the provided extra sheets. This serves as an introduction to **compartmental models**, which we will study more next class.

2. In this question, we introduce the **mass-spring system**, which we will return to frequently throughout the semester.

Imagine we have a spring attached to the wall on one end, and to a cart on the other, as in the picture below



In order to be able to predict the behavior of the cart, our system records position (X) and velocity (V).

(a) What is the state space of this system?

(b) How are position and velocity related?

(c) How are velocity and acceleration related? Use your answer to write the change equation for the velocity in terms of force and mass. (Hint: This involves Newton's second law $F = ma$)

(d) According to Hooke's law, the force of a spring is proportional to how stretched out it is with a proportionality constant k (called the "spring constant"). The direction of the force is opposite to the displacement (in other words, the force is directed toward the "rest position" of the spring). Write down Hook's law and then use it to write the change equations of the system.

(e) Is there a feedback loop present in the system? Explain why or why not.

(f) With the assumptions we have made, how will the position (and velocity) of the cart evolve with time?

(g) A more accurate model should account for friction, whether it's from air resistance, the surface resistance against the wheels moving, etc. Suppose that friction is a force that is opposite to the movement of the cart, and proportional to its velocity. Write the change equations for the mass-spring model with friction (you can make up parameters if necessary).