Recitation 3

1. On a hot day you go to the Cornell dairy bar to get an ice cream. You notice that:

- People join the line at a rate proportional to the number of people already in the line, with a proportionality constant of 0.1 (ie. the more people waiting, the more people passing by think about buying an ice cream).
- People order, get their ice cream and leave at a constant rate of 0.4 per minute.
- If people wait for too long they get tired of standing in line and leave at a per capita rate proportional to the number of people in line, with a proportionality constant of 0.02.

Build model following the steps we have seen in class.

(a) What is the system?

(b) What is the state variable. What are its units.

(c) What is the state space?

(d) What is the *word equation (ie. the change equation(s) in words)? Specify the inflows and outflows.*

(e) What is the change equation (in mathematical symbols)?

2. In the reading for today, you learned about the population model with crowding $X' = bX\left(1 - \frac{X}{k}\right)$. The goal of this exercise is to see why this model makes sense.

(a) Why do we need to introduce a crowding term in the population model?

(b) Suppose $b = 0.1$ and $k = 100$. What is the sign of X' when X is equal to 50, 100, 200?

(c) For what values of X is the sign of X' positive/negative? When is X' zero?

(d) Now, consider the general model $X' = bX\left(1 - \frac{X}{k}\right)$. For what values of X is the sign of X' positive/negative? When is X' zero?

(e) For the general model, describe in words when the population is increasing or decreasing or staying the same. Use the fact that X' positive means that X increases, X' negative means X decreases and $X' = 0$ means no change.

3. For the following situations, indicate the state variable, its units and the state space.

(a) The weight of a person

(b) The velocity of an object

Supplemental Questions

A. We want to build a model for the temperature of a cup of tea. We can imagine that the tea is hot and the room is cooler than the tea. According to Newton's law of cooling, the change in the tea temperature is proportional to the difference between the temperature of the tea and the temperature of the room. Build model following the steps we have seen in class.

(a) What is the system?

(b) What is the state variable. What are its units?

(c) What is the state space?

(d) What is the word equation?

(e) What is the change equation (in mathematical symbols)?