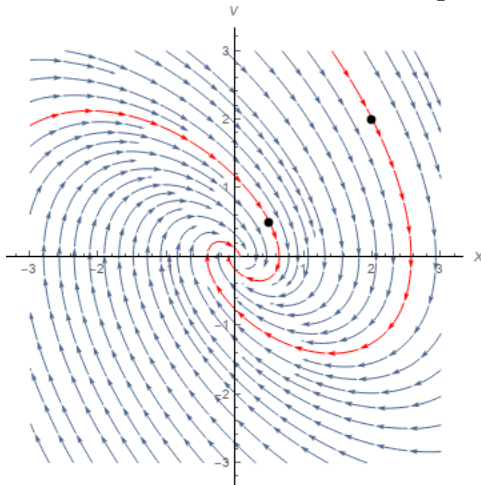


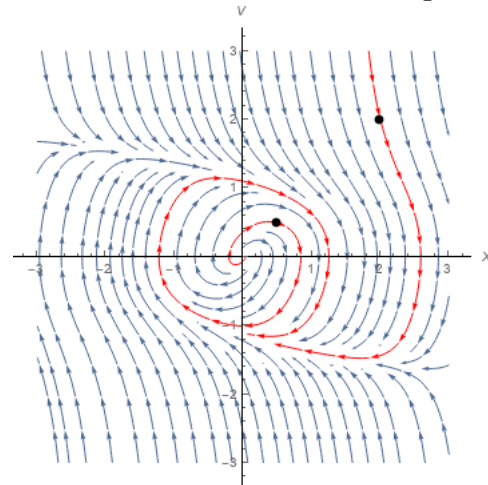
Recitation 25

1. Previously, we have seen that Rayleigh's model for the clarinet reed constitutes an example of Hopf bifurcation. Below are the trajectories we looked at. For both cases, sketch the time series starting at $(0.5, 0.5)$ and $(2, 2)$. As a reminder, the important characteristics of your time series are the relative locations and values of the extrema and the general shape/patterns of the curves.

Clarinet Reed Without Blowing



Clarinet Reed With Blowing



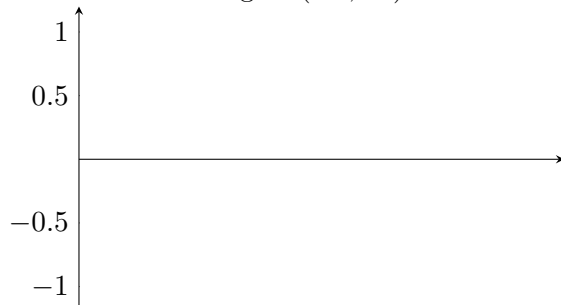
Starting at $(2, 2)$:



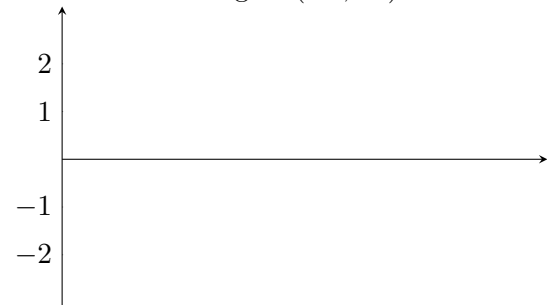
Starting at $(2, 2)$:



Starting at $(0.5, 0.5)$:



Starting at $(0.5, 0.5)$:



2. What is the main difference between the bifurcation diagram for the lac operon and the bifurcation diagram for the spruce budworm system? Where does the difference come from?

3. How many bifurcation points does the lac operon system have? What type of bifurcations are they? How about the spruce budworm system?

4. List all the types of bifurcation we have seen and describe each of them in words. Then, for each type, give at least one example and indicate whether the associated bifurcation diagram shows the equilibrium points or if it shows the bifurcation curves.