Recitation 24

In today's recitation, we will explore some of the limitations of the Lotka-Volterra population model. By correcting these limitations, we will arrive at another population model, the Holling-Tanner model.

The Lotka-Volterra model for predator (P) and prey (N) populations is given by

$$N' = bN - aNP$$
$$P' = maNP - dP$$

1. Prey Birth Rate:

(a) Consider the prey equation. In the absence of predators ($P = 0$), what does the model predict about the prey population? Is this realistic.

(b) By what could we replace the term bN to have something more realistic?

2. Next, we consider the term that corresponds to a predator encountering and eating a prey.

(a) Which term is this?

(b) Suppose that there is a fixed number of predators. What happens to this term as N grows very large? Is this realistic?

(c) We would like to put an upper limit to the number of preys a single predator can eat per unit of time. Therefore, we'd like to find a function that starts at 0, increases and then levels off. Which function that we have studied fits this description? (Hint: look at the graphs at `https://www.desmos.com/calculator/6fwjdtu4ep`.)

(d) In this situation it makes sense to say that the prey consumption grows really fast at first. Which value of n should we choose to best model this?

(e) We will make a small change to this function and replace the 1 in the denominator by a parameter d. This parameter is often called "half-saturation density". Where does the name come from? To answer this question, look at the value of the function when $X = d$.

(f) Using this function, write the new term "predator eats a prey". Use w for the leading coefficient.

3. Using your answers from Problems 1 and 2, write the modified prey equation.

4. Next, we consider the predator equation.

Instead of having separate birth and death terms, we will assume that the predator population is governed by a logistic equation, with per capita birth rate and with an environmental carrying capacity dependant on the on the number of preys, N, (rather than on a fix parameter k).

(a) Write down a "normal" logistic growth equation for the population of P predators with birth rate r_2 and carrying capacity k.

(b) For which values of P does the population increase? For which values of P does the population decrease?

(c) Let j be the number of prey needed to support one predator. How many prey are needed to support a population of P predators?

(d) If the number N of prey is smaller than your previous answer, should the predator population increase or decrease? What if N is larger than your previous answer?

(e) Comparing your answers from parts (b) and (d), notice that $\frac{N}{j}$ plays the same role as k, so we may use this as our carrying capacity. Use this to write the modified predator equation.

5. In summary:

(a) What were the two limitations of the Lotka-Volterra model?

(b) What are the change equations of the Holling-Tanner model? What do each of the 6 free parameters represent?