Recitation 23

1. The logistic equation predicts that when a small population is introduced to a new habitat, it will smoothly grow until reaching carrying capacity and then level off. However, what we often observe in such cases is an overshoot and collapse pattern, in which the population grows to a high density and then crashes. We can attribute this to a time delay.

(a) Write a change equation for the population of a species (N) with a logistic birth rate and an additional per capita death rate d.

One simplification of the models we have looked at so far is that all members of a population contribute to its reproductive dynamics. However, this is clearly not the case; there is a period of time before which an individual reaches sexual maturity.

The rest of this question focuses on how we can adapt the above model to account for this.

(b) We will introduce an *auxiliary* state variable A to keep track of the adolescent (incapable of reproducing) population, so that the original state variable N can model the adult (capable of reproducing) population. Write the change equation for A assuming a logistic birth rate of the adult population. (Be sure to explicitly introduce a time dependence).

(c) Assume that it takes time τ for a newborn individual to mature to adulthood. How can we relate A' and N', assuming that all individuals mature to adulthood, and that the per capita death rate of adults is d.

(d) Finally, combine your answers from parts (b) and (c) to eliminate the auxiliary variable, leaving a change equation depending only on the original state variable N.

2. The garibaldi is a large orange fish that lives off the coast of California. Adult garibaldis lay eggs, a fraction of those eggs hatch, and only a portion of those larval garibaldis survive to adulthood.

Here, we will write a differential equation for the size (G) of the adult garibaldi population, based on a list of assumptions. Just as in question 1, it will be helpful to introduce auxiliary variables : E for the number of eggs, and L for the number of larvae.

Express each of the following assumptions as a differential equation (or a term of a differential equation). Then, combine these equations to find a differential equation for G.

- Garibaldis lay eggs at a per capita rate b.
- Since garibaldis sometimes eat their own eggs, the fraction of eggs that hatch is a decreasing sigmoid function of the adult population.
- Larval garibaldis float as plankton before becoming adults and joining a population. Thus, the number of individuals joining a population is proportional to the number that hatched six years earlier, with proportionality constant r.
- Adult garibaldis die at a per capita rate d.

3. Imagine you are driving on a highway and you have just made your way out of a very frustrating traffic jam. You look for the reason of the traffic jam such as an accident or road work but there is nothing. This is known as a "phantom traffic jam" [1]

Let $D(t)$ as the distance between your car and the car preceding you. And let d_O be the optimal distance between two cars. Use the concepts seen in class to explain the causes of the phenomenon and suggest ways it could be fixed.

[1] You can see examples of such "phantom traffic jams" in these videos: `https://youtu.be/Suugn-p5C1M` and `https://youtu.be/Rryu85BtALM`.