$\rightarrow t$ 

## Recitation 22

- 1. In this question, we determine what the time series of a system with a periodic attractor looks like.
  - (a) Consider the Holling-Tanner prey-predator model. In the lecture we sketched two trajectories we get after a predator removal. Pick one of these trajectories (redraw it on the vector field below) and sketch the times series associated to this trajectory<sup>1</sup>.



(b) Do the same for the Lotka-Voltera model.



(c) What is the main difference between these two time series? (What is the main characteristic of each time series and how are they different from each other)?

 $<sup>^{1}</sup>$ Do not be too concerned about the exact shape of the trajectory. Instead focus on the extreme values of each population and the general patterns of the time series.

2. Does every trajectory that approaches a limit cycle attractor ever reach the attractor?

**3.** If a periodic attractor contains a single equilibrium point (and no other periodic attractor) inside itself, what kind of equilibrium must the point be? (Hint: try sketching trajectories inside of the limit cycle for different types of equilibria.)

**4.** A periodic attractor is also called a *stable limit cycle* (because if you are pushed slightly away from the limit cycle, you get back to it). Sketch a phase portrait of an *unstable limit cycle*.

5. If the unstable limit cycle has a single equilibrium point (and no other limit cycles) inside itself, what kind of equilibrium must the point be?