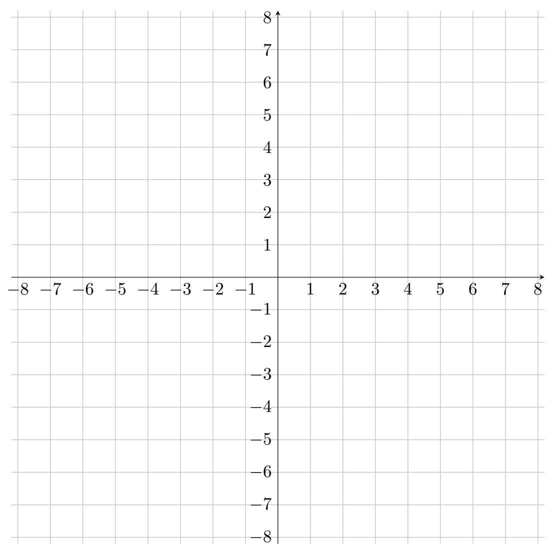
Recitation 2

1. Use what you've learned about addition and scalar multiplication of vectors to compute the following:

1. for $\mathbf{a} = (0, 3)$ and $\mathbf{b} = (-2, -1)$, compute $\mathbf{a} + \mathbf{b} =$
2. for $\mathbf{a} = (5, -3)$ and $\mathbf{b} = (2, 4)$, compute $\mathbf{a} - \mathbf{b} =$
3. for $\mathbf{a} = (2, -1)$ and $\mathbf{b} = (-2, 4)$, compute $4\mathbf{a} + \frac{1}{2}\mathbf{b} =$
4. or $\mathbf{a} = (3, 2, 1)$ and $\mathbf{b} = (2, 5, 3)$, compute$-2\mathbf{a} + 3\mathbf{b} =$

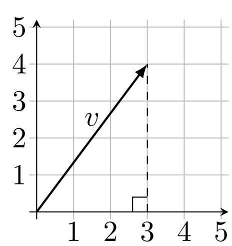
For parts 1,2, and 3, draw each vector, together with the resulting vectors, on the graph below. Note that we can make the first vector start wherever we want.



Would it be possible to draw the vectors of part 4 here too? Explain why or why not.

2. In this question, we explore the notion of a vector's **norm** (also called its **magnitude** or **length**).

(a) First, we consider the vector $v = (3, 4)$. Based on the diagram below, how can we compute the length of this vector (notated as $||v||$)?



(b) Use your intuition from part (a) to write down a general formula for the length of a 2-component vector (a, b). Then, use this formula to exactly compute the lengths of the following vectors.

$$||(5, 12)||$$

$$||(-8, 15)||$$

$$||(-2, -6)||$$

(c) The formula for the length of a 3-dimensional vector is given by

$$||(a, b, c)|| = \sqrt{a^2 + b^2 + c^2}.$$

If you are interested in why this is the case, we walk through an argument in the Supplemental Exercises. Use this formula to exactly compute the lengths of the following vectors.

$$||(2, 1, 2)||$$

$$||(-4, 12, -3)||$$

$$||(7, -6, 1)||$$

(d) In general, what effect does multiplying a vector by a scalar s have on its length. (Compare the length of a general vector (a, b) with the length of vector $s(a, b)$.)

(e) A vector with length 1 is called a **unit vector**. Unit vectors are useful in situations where we care about the direction of a vector, but want to disregard its length (we will see this later).

To **normalize** a vector is to scale it so that it has length 1. From part (d), we see that this is accomplished with scalar multiplication by $\frac{1}{||v||}$. Normalize the following vectors.

$$(24, 7)$$

$$(-1, 2)$$

$$(4, 7, -4)$$

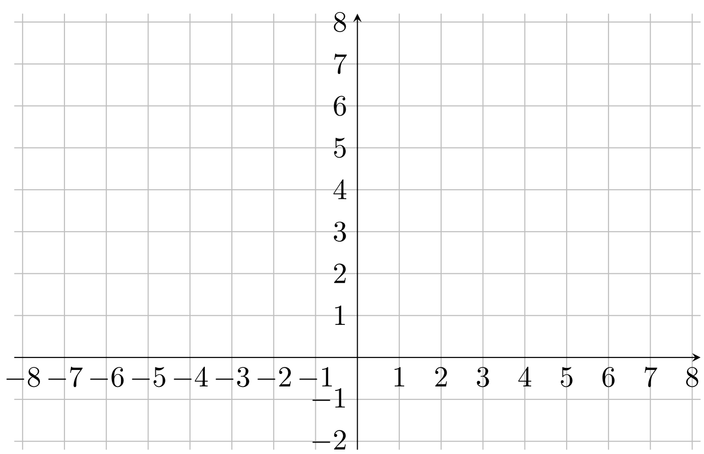
Supplemental Questions

A. Imagine you are taking part in a treasure hunt. The treasure is buried in a field and the instructions to find it are given in terms of vectors. The instructions are:

1. Starting at the big oak tree, follow $(3, 2)$.
2. Follow $(1, 2)$ twice.
3. Follow $(-5, 0)$.
4. Follow $(-6, -4)$ for half its length.
5. Start digging!

(a) Write down the list of points where you have stopped to change direction.

(b) On the graph below, draw the vectors and points where you have stopped. Assume the big oak tree is at $(0, 0)$.

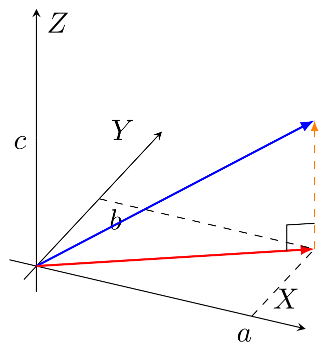


(c) Where is the treasure buried?

(d) How far is the treasure from the big oak tree?

B. In this question, we derive the formula for the length of a 3-component vector, given above.

Consider a general 3-component vector $v = (a, b, c)$. We can plot v using a 3-dimensional plot, as shown in blue in the picture below (here, we have made a, b, c all positive, but this is not necessary).



The red vector is $(a, b, 0)$, the **projection** of the blue vector onto the XY-plane (that is, the shadow that the blue vector casts onto the XY-plane).

(a) Use your formula from part (a) to compute the length of the red vector.

(b) What is the orange vector? What is its length?

(c) Finally, notice that the blue, red, and orange vectors form a right triangle. Use your answers from parts (a) and (b) to find a formula for the length of the blue vector.

(d) As is often the case, once we found a way to "generalize" our formula for vector length to 3-components, we can apply the same ideas to generalize it to even more components. Based on this, what would you suspect the length of the vector $(1, 2, 3, 4)$ to be?

Take a second to appreciate this, because it's really cool. You just used math to measure something in 4-dimensional space, something that we could never tangibly do in our 3-dimensional world!