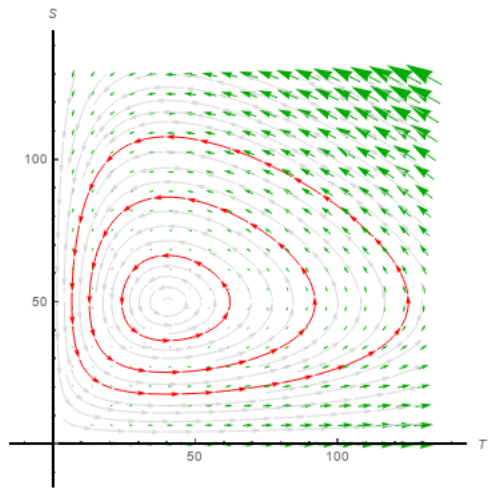
Recitation 18

1. Consider the Tuna-Shark system given by the equations:

$$T' = 0.5T - 0.01ST$$
$$S' = 0.005ST - 0.2S$$

(a) Verify that $(0, 0)$ and $(40, 50)$ are the two equilibrium points of this system.

(b) Let us now focus on the equilibrium point $(40, 50)$. Use the vector field and trajectories shown below to determine which type of equilibrium point it is.



(c) At any given point in the state space, describe the trajectory followed by the system.

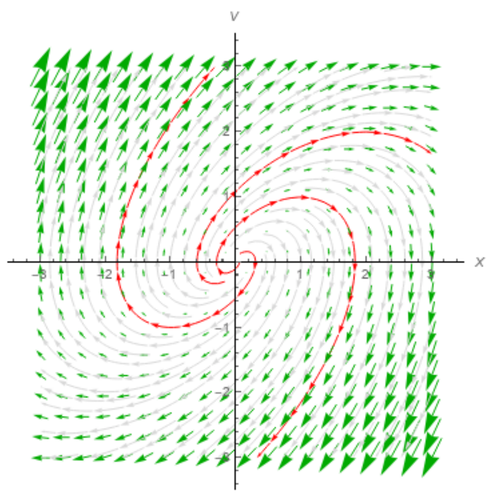
(d) What is the long term behavior of this system (and more generally the long-term behavior associated with this type of equilibrium point)? *Hint: think about the time series associated to a trajectory.*

2. Consider the mass-spring system. Let us assume that instead of having "normal" friction, a "negative" friction is applied to it. Concretely, a negative friction could be an outside force pushing or pulling the mass in the same direction as velocity [1]. The equations are in this case are

$$X' = V$$
$$V' = -X + V$$

(a) Find the equilibrium point of this system. What does this point correspond to in terms of the system?

(b) The vector field and the trajectories of this system is shown below. Determine the type of equilibrium of the equilibrium point. Explain your reasoning.



(c) At any given point in the state space, describe the trajectory followed by the system.

(d) What is the long term behavior of this system (and more generally the long-term behavior associated with this type of equilibrium point)? *Hint: think about the time series associated to a trajectory.*

[1] This type of oscillation is often called *forced oscillation. For more information, see* `https://youtu.be/hbjEAG2eSp8?t=69`. *Forced oscillations and resonance are a known problem for bridges, such as in the famous collapse of the Tacoma Narrows bridge.*

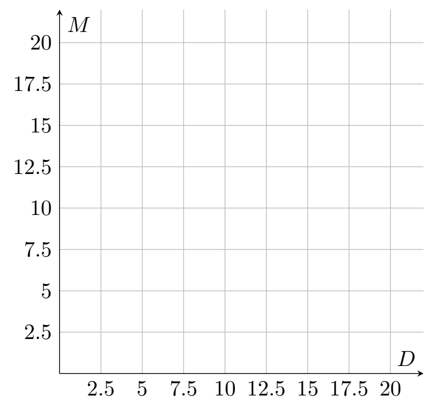
3. Be Sure To Have This Exercise at Hand Next Class.

Next class we will determine the type of equilibrium points in deer-moose competition models. To do so, we will have to compute the equation of lines in the DM-plane (ie. D is the horizontal axis and M the vertical one). To be ready, we will do some of these computations here.

(a) Consider the equations

$$0 = 15 - 2M - D$$
$$0 = 10 - M - D$$

Rewrite these equations in the slope-intercept form, draw these lines in the graph below, and determine their intersection point.



(b) Repeat the process for the equations

$$0 = 15 - M - D$$
$$0 = 10 - M - 0.5D$$

Rewrite these equations in the slope-intercept form, draw these lines in the graph below, and determine their intersection point.