Recitation 15

1. Suppose that $f'(X) = \cos(e^X) + X^2 \ln(X)$ (note that this is a continuous function on $(0, \infty)$).

(a) Use the Fundamental Theorem of Calculus to write down a general formula for $f(X)$.

(b) Further suppose that $f(3) = 0$. How can we modify our general formula to account for this?

2. How can we verify that $F(X) = \frac{1}{22}(X^2 + 3)^{11} + 7$ is an antiderivative of $f(X) = X(X^2 + 3)^{10}$ without explicitly computing the antiderivative?

As we discussed in class, an **antiderivative**, sometimes called an **indefinite integral**, of a function $f(X)$ is a function $F(X)$ such that $F'(X) = f(X)$. We notate this as an integral without bounds

$$\int f(X)dX = F(X).$$

Thus, a indefinite integral reverses the process of taking a derivative (so their computation involves "reverse engineering" the differentiation rules we discussed in class).

3. Compute the following indefinite integrals, being sure to include a constant of integration.

For the last two integrals, do not try to reverse engineer the product/quotient rules. Rather, manipulate the inside of the integrals until they are sums of functions that you know how to handle.

(a) $\displaystyle\int X^4 + X + 5 + e^X \, dX$

(b) $\displaystyle\int \sin(X) + 2\cos(X) \, dX$

(c) $\displaystyle\int 7X^2(2X + 6) \, dX$

(d) $\displaystyle\int \frac{4X^3 - 2X - 5}{X^2} \, dX$

4. Use the Second Fundamental Theorem of Calculus to compute the following definite integrals.

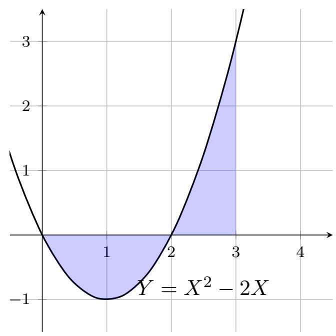
(a) $\displaystyle\int_0^3 5X^2 - 6X + 3 \, dX$

(b) $\displaystyle\int_1^2 \frac{8}{X^2} \, dX$

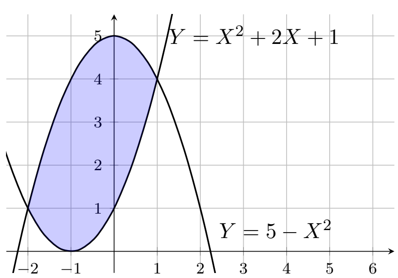
(c) $\displaystyle\int_0^\pi 2\cos(X) + 3 \, dX$

5. Find the areas of the following shaded regions. Remember that the integral computes a "signed" area.

(a)



(b)



Supplemental Questions

This question is tricky, and introduces concepts that we don't expect you to know. Saying this, it is very interesting mathematically, so it is worthwhile if you'd like to learn a little more about integration.

A. In multivariable calculus, one often needs to calculate an **iterated integral**, which generalizes the notion of "area under the curve" to the volume enclosed by a series of curves. One crucial technique in evaluating these iterated integrals involves changing coordinate systems. In this question, we'll develop some of these ideas.

(a) Let $f(X)$ and $g(X)$ be continuous functions, and define the function $h(X)$ by

$$h(X) = \int_0^{g(X)} f(t)dt.$$

Write an expression for $h'(X)$. (Hint: Apply the Fundamental Theorem of Calculus and the Chain Rule. This function will involve f, g, and (possibly) their derivatives.)

(b) Use the formula that you found in part (a) to compute $h'(X)$ when

$$h(X) = \int_0^{\ln(X)} te^t \, dt.$$

Simplify your answer.

(c) Argue that $h(X) = X\ln(X) - X + 1$. (Hint: Show that the derivative of $h(X)$ agrees with your answer from part (b), and that $h(1) = 0$, both here and in the integral expression above.)