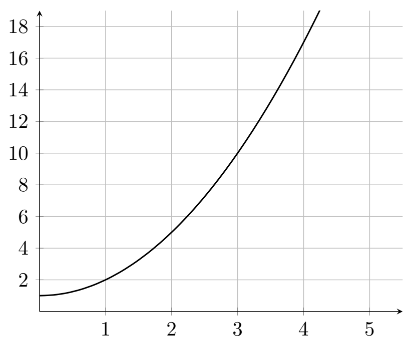
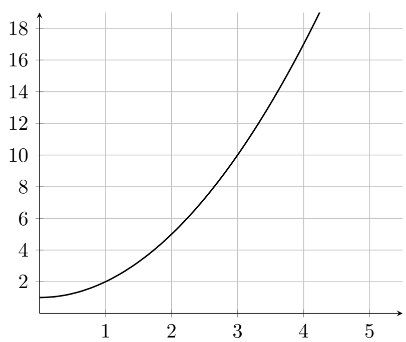
Recitation 14

1. In this question, we approximate the area under the graph $f(X) = X^2 + 1$ between $X = 1$ and $X = 4$ using different step sizes.

(a) First, compute this area using a (left) Riemann sum with step size $\Delta X = 1$. Draw the approximation on the graph below, and indicate whether it is an under- or over-approximation.



(b) Repeat the above using step size $\Delta X = \frac{1}{2}$.

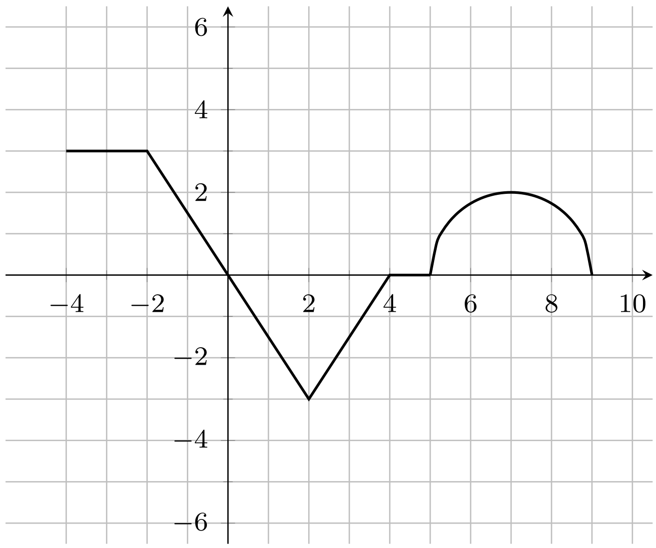


The actual value of this area is 24. We will see how to calculate this next class.

(c) Which of these Riemann approximations is better? How can you explain this using your graphs?

(d) What can we do to make our approximation even better.

2. Consider the graph of a function $g(X)$, given below.



Compute the following integrals. (Recall that the integral is a measure of the *signed* area between the graph and the X-axis. Also, remember that the area of a circle with radius r is πr^2.)

(a) $\int_{-4}^{0} g(X)dX$

(b) $\int_{0}^{4} g(X)dX$

(c) $\int_{5}^{9} g(X)dX$

(d) $\int_{-4}^{9} g(X)dX$

(e) $\int_{-2}^{2} g(X)dX$

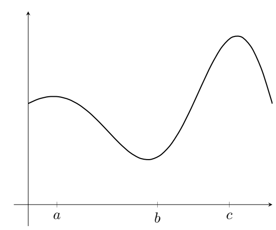
(f) $\int_{-2}^{2} |g(X)|dX$

(g) $\int_{-4}^{9} |g(X)|dX$

3. In this question, we consider some rules for the bounds of integrals. Throughout, we assume that $a \leq b \leq c$ are constants and $f(X)$ is an arbitrary function.

(a) How would we interpret $\int_a^a f(X)dX$ as an area (take note of the bounds). What is the value of this integral?

(b) Express $\int_a^c f(X)dX$ in terms of $\int_a^b f(X)dX$ and $\int_b^c f(X)dX$. To do this, interpret each of these integrals as an area, and sketch them on the example graph below.



Note that this rule holds more generally for any a, b, c.

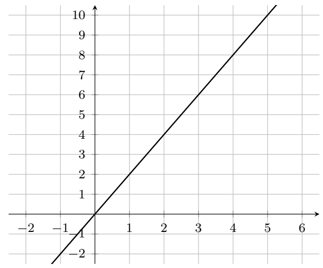
(c) Rewrite your answer from part (b) to express $\int_b^c f(X)dX$ in terms of $\int_a^c f(X)dX$ and $\int_a^b f(X)dX$.

Notice that the integrals on the right side of this expression have the same lower bound, or **base point**. We will use this idea in our proof of the Second Fundamental Theorem of Calculus next class.

(d) How can we express $\int_b^a f(X)dX$ in terms of $\int_a^b f(X)dX$? (Hint: use parts (a) and (b). This is a little tricky.)

Supplemental Questions

A. In this question, we consider the function $h(X) = 2X$.



(a) What is $\displaystyle\int_0^2 h(t)dt$?

(Note that we switched the inner variable from X to t. This doesn't change the computation, but is important for reasons we will see later.)

(b) What is $\displaystyle\int_0^5 h(t)dt$?

(c) More generally, what is $\displaystyle\int_0^X h(t)dt$?

(Note that there is a variable, X, in the upper bound, so your answer should be a function that takes in an X-value and returns an area.)

(d) Is there a relationship between your answer to part (c), which we call $H(X)$, and $h(X)$? If so, what is it?

We will build upon the ideas from this problem next class when we study the Fundamental Theorem of Calculus.