Recitation 12

1. For each of the following functions, compute the derivative in 2 ways:

- Using the product/quotient/chain rules.
- Expanding/simplifying the function first, then using the power rule.

Make sure your answers match. Make a note of which method was easier for each question.

(a) $f(X) = (X^3 + 6)(X^2 - 3X)$

(b) $f(X) = \dfrac{X^2 + X - 6}{X - 2}$

(c) $f(X) = (X - 4)^3$

2. In this question, we draw connections between the graph of a function and the graph of its derivative.

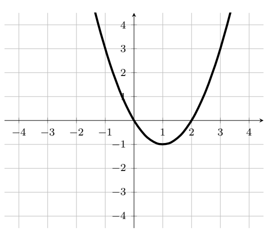
(a) The derivative, a function itself, tells us the (instantaneous) slope of the function at that X-value. What is the significance of the sign of the derivative (in other words, what does the function look like when its derivative is positive/negative).

(b) What is the derivative at a local minimum/maximum (sometimes called a local extremum) of a function. (It may help to draw a small sketch.)

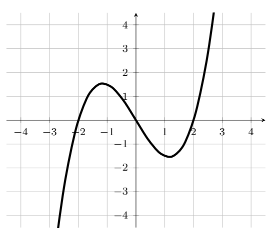
(c) Use the ideas from parts (a) and (b) to match the functions to their derivatives.

Functions:

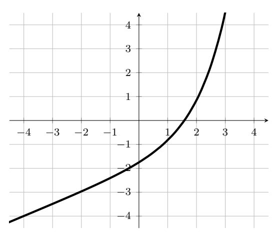
A.



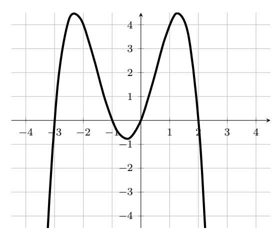
B.



C.

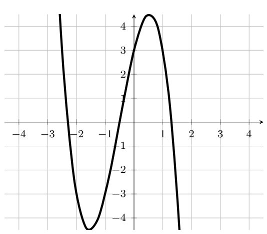


D.



Derivatives:

1.



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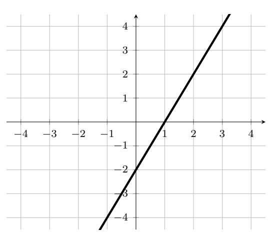

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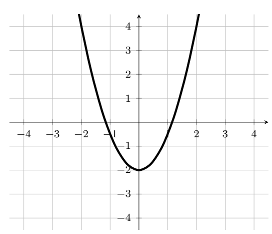

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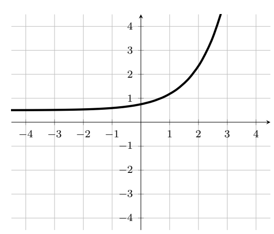

3. Compute the derivatives of the following functions.

(a) $f(X) = \sin^2(X) - \sin(X^2)$

(b) $g(X) = \dfrac{\ln(X)}{X}$

(c) $h(X) = \ln\left(\dfrac{1}{X}\right)$

(d) $k(X) = X^2 e^X \cos(X)$

(e) $\ell(X) = e^{e^X}$

(f) $m(X) = \sin(X)\cos(X)$

Supplemental Questions

A. From the reading, we know that the derivative of $\cos(X)$ is $-\sin(X)$. In this exercise, we give a proof of this using the Pythagorean trig identity $\sin^2(X) + \cos^2(X) = 1$.

(a) Rearrange this identity to solve for $\cos(X)$ in terms of $\sin(X)$.

(b) Now, take the derivative of this function of $\sin(X)$. (This will involve multiple applications of the chain rule.)

(c) Finally, use part (a) again to simplify the derivative to $-\sin(X)$.

In a similar manner, we can use different trig identities to solve for the derivatives of the remaining trig functions.

B. In this exercise, we give a derivation of the quotient rule. We can rewrite the quotient of two functions:

$$q(X) = \frac{f(X)}{g(X)} = f(X) \cdot (g(X))^{-1}.$$

Use the product rule and the chain rule to compute $q'(X)$.