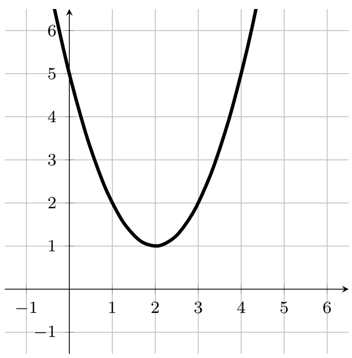
Recitation 11

1. (a) Using the limit definition, compute the derivative of $f(X) = X^2 - 4X + 5$, shown below.



(b) Compute $f'(2)$. How does this value relate to the graph shown above?

Repeat this for $f'(1)$ and $f'(4)$.

2. While it is possible to apply the limit definition to compute the derivative of any function, it is often very impractical. Instead, we use derivative rules, which we will learn over the next few classes, to make the computations simpler.

The first rule, which we learned last class, is the **power rule**. Given a function of the form $f(X) = X^n$ for any constant n, its derivative is given by

$$f'(X) = nX^{n-1}.$$

Use the power rule to compute the derivatives of the following functions.

(a) $f(X) = X^3$

(b) $f(X) = X^{1043}$

(c) $f(X) = \dfrac{1}{X}$

(d) $f(X) = \sqrt{X}$

(e) $f(X) = \dfrac{1}{\sqrt[3]{X}}$

3. Here, we explore some facts that we will use to take more complicated derivatives [1].

(a) First, we show that derivatives preserve addition. Suppose that the function $s(X)$ is defined as the sum $s(X) = f(X) + g(X)$. Show, using the limit definition of the derivative, that $s'(X) = f'(X) + g'(X)$.

(b) Next, we show that derivatives preserve scalar multiplication. Suppose that $m(X) = c \cdot f(X)$ for some constant c. Show, using the limit definition of the derivative, that $m'(X) = c \cdot f'(X)$.

[1] These facts show that derivatives are **linear operators**; they preserve the operations of addition and scalar multiplication. Recall that these are the same operations that we have defined on vectors. The study of these operations on various mathematical objects, known as **vector spaces**, is the focus of **linear algebra**.

4. Use the facts from Exercise 3 to compute the derivatives of the following functions:

(a) $f(X) = X^2 - 4X + 5$

(b) $f(X) = 2\sqrt{X} - \dfrac{1}{2}X^2$

(c) $f(X) = 6X^6 + 5X^5 + 4X^4 + 3X^3 + 2X^2 + X$

(d) $f(X) = \dfrac{1}{X} + \dfrac{1}{2X^2} + \dfrac{1}{3X^3}$

(e) $f(X) = X^\pi - X^e + e^\pi$

Check to make sure that your answer to part (a) agrees with your answer to Question 1.

Supplemental Questions

A. Using the limit definition, compute the derivative of $g(X) = \frac{X}{X+2}$.