Simple yet Efficient Estimators for Network Causal Inference Even When the Network is Unknown

Mayleen Cortez, Matthew Eichhorn, Christina Lee Yu

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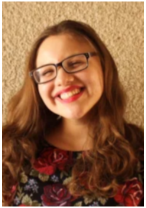
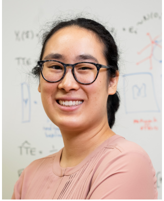




Our Goal

Predict the effect of exposing an entire population to a treatment

- How will a national advertising campaign influence sales?
- How will a new website feature impact user engagement?
- Will proposed public health measures mitigate the spread of a disease?

https://www.cbia.com/resources/coronavirus/reopen-connecticut/covid-19-poster-slow-the-spread/

Total Treatment Effect

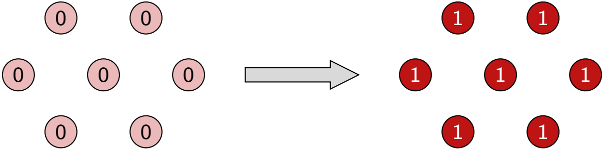
Population: Individuals $\{1, \ldots, n\}$

Treatment: $\mathbf{z} \in \{0,1\}^n$, $1 =$ treated , $0 =$ untreated

Outcomes: Each individual i has function $Y_i : \{0,1\}^n \to \mathbb{R}$

The *Total Treatment Effect* is the average effect on an individual's outcome if entire population is treated.

$$TTE = \frac{1}{n} \sum_{i=1}^{n} \Big(Y_i(\mathbf{1}) - Y_i(\mathbf{0}) \Big).$$



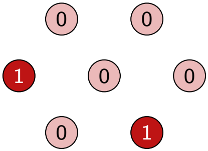
Estimating TTE

We wish to estimate TTE after treating a small fraction of individuals

- Experimental trials can be costly or resource intensive
- Treated individuals could see permanent worsened outcome

Classical Approach:

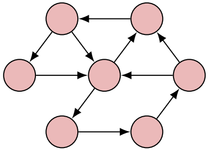
- Treat a random subset of individuals, and scale appropriately
- Difference of means estimator
- Relies on SUTVA: each Y_i is a function only of z_i

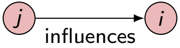


Incorporating Network Effects

Real-world applications include interactions between individuals.

- People purchase goods based on recommendations from friends
- Engagement on a social platform is affected by how engaged one's connections are
- Public health policy in one community affects disease transmission to neighboring communities





Network effects violate SUTVA, introduce bias into classical estimators.

Potential Outcomes with Network Effects

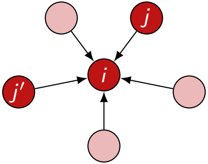
Neighborhood Interference:

Y_i is a function of $\{z_j : j \in \mathcal{N}_i\}$

Low Degree (β):

Subsets $\mathcal{S} \subseteq \mathcal{N}_i$ of treated neighbors ($|\mathcal{S}| \leq \beta$) realize effects $c_{i,\mathcal{S}}$ in Y_i

$$Y_i = c_{i,\varnothing} + \sum_{\substack{\mathcal{S} \subseteq \mathcal{N}_i \\ 1 \leq |\mathcal{S}| \leq \beta}} c_{i,\mathcal{S}} \prod_{j \in \mathcal{S}} z_j$$



$\beta = 1$: Linear Heterogeneous Outcomes Model
$\beta = |\mathcal{N}_i|$: Completely General

Techniques for Network Causal Inference

Existing Approaches:

Difference of Means: Biased

OLS: Works well for uniform treatment effects $c_{i,\mathcal{S}}$, otherwise biased

Horwitz-Thompson/Hájek: Unbiased, but require treatment / non-treatment of entire neighborhoods (clustering helps, but is costly)

Our Contributions:

Known Network Structure: Unbiased estimator that performs well under *Bernoulli randomized design*

Unknown Network Structure: Unbiased estimator under *staggered rollout Bernoulli design* based on polynomial extrapolation

Graph-Aware Estimator

$$\widehat{TTE} = \frac{1}{n}\sum_{i=1}^{n} Y_i(\mathbf{z}) \sum_{\substack{\mathcal{T}\subseteq\mathcal{N}_i \\ |\mathcal{T}|\leq\beta}} f_{|\mathcal{T}|} \prod_{k\in\mathcal{T}} \left(\frac{z_k}{p} - \frac{1-z_k}{1-p}\right), \qquad f_{|\mathcal{T}|} = (1-p)^{|\mathcal{T}|} - (-p)^{|\mathcal{T}|}$$

Theorem

$\widehat{TTE}$ is unbiased with variance $O\left(\frac{d^2 Y_{\max}^2}{n} \max\left(\left(\frac{2\beta}{p}\right)^{2\beta}, \left(\frac{d}{p(1-p)}\right)^{\beta}\right)\right)$.

Ingredients for Unbiasedness:

- Lots of linearity of expectation
- Identity $\mathbb{E}\left[\prod_{j\in\mathcal{S}} z_j \prod_{k\in\mathcal{T}} \left(\frac{z_k}{p} - \frac{1-z_k}{1-p}\right)\right] = \mathbb{I}(\mathcal{T}\subseteq\mathcal{S}) \cdot p^{|\mathcal{S}|-|\mathcal{T}|}$
- Binomial theorem (using $f_{|\mathcal{T}|}$s) to remove $c_{i,\varnothing}$

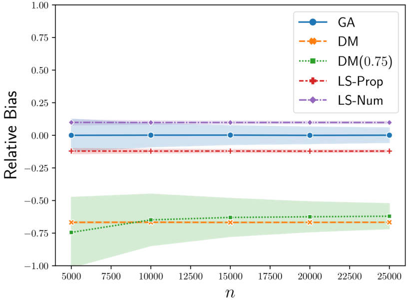
Graph-Aware Estimator in Practice

Erdős-Rényi directed networks with varying n, edge probability $\frac{10}{n}$

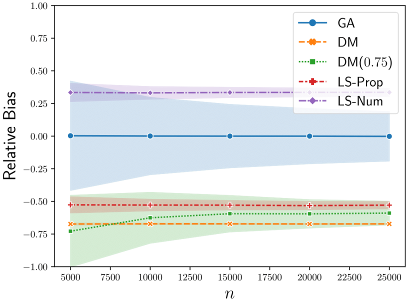
Randomly sampled $\{c_{i,\mathcal{S}}\}$: ~33% magnitude from direct effects $\{c_{i,\{i\}}\}$

Overall treatment probability $p = 0.2$

Linear



Quadratic



Graph-Agnostic Estimator

Staggered-Rollout: Assign treatment to individuals over $\beta+1$ stages: $\mathbf{z}^0 \preceq \mathbf{z}^1 \preceq \ldots \preceq \mathbf{z}^\beta$, with $\mathbf{z}^t \sim \mathrm{Bern}(p_t)$

Measure outcomes $Y_i(\mathbf{z}^t)$ after each stage

Recast estimation as polynomial extrapolation problem:

Learn $F(p) = \mathop{\mathbb{E}}_{\mathbf{z}\sim\mathrm{Bern}(p)}\left[\frac{1}{n}\sum_{i=1}^{n} Y_i(\mathbf{z})\right]$ to calculate $\mathrm{TTE} = F(1) - F(0)$

Estimator based on Lagrange interpolation:

$$\widehat{TTE} := \frac{1}{n}\sum_{i=1}^{n}\sum_{t=0}^{\beta}\Big(\ell_t(1) - \ell_t(0)\Big)\cdot Y_i(\mathbf{z}^t), \qquad \ell_t(p) = \prod_{\substack{s=0 \\ s\neq t}}^{\beta} \frac{p-p_s}{p_t-p_s}$$

Theorem

When $p_t = \frac{tp}{\beta}$, $\widehat{TTE}$ is unbiased with variance $O\left(\frac{d^2\beta^2}{n}\cdot Y_{\max}^2\cdot\left(\frac{\beta}{p}\right)^{2\beta}\right)$.

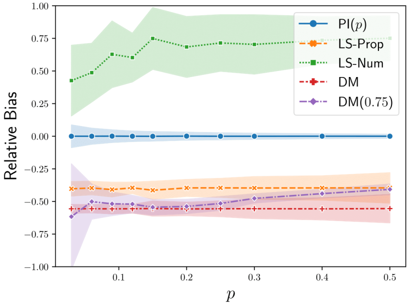
Graph-Agnostic Estimator in Practice

Random directed network on 5000 individuals using configuration model

- In-degrees distributed according to power law ($\alpha = 2.5$)
- Out-degrees are consistent

Randomly sampled $\{c_{i,\mathcal{S}}\}$: ∼44% magnitude from direct effects $\{c_{i,\{i\}}\}$

Linear



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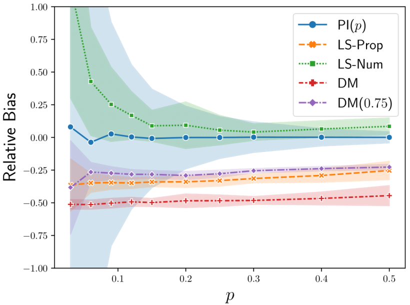

Thank You!